STOCHASTIC PROCESSES

Exercises for Lecture 1

Exercise 1. Consider a population made of two subject groups, *A* (healthy subjects) and *B* (subjects with a pathology), and an automatic diagnostic system that classifies the subjects into *A* and *B*. The probability that a subject belongs to group *A* is P(*A*) = 0.95. The probabilities of error in the classification by the automatic system for the two groups are P(*E*|*A*) = 0.1 and P(*E*|*B*) = 0.001. Compute the probability of error on the population.

Solution

If $B_1, B_2, \ldots, B_m$ are a set of mutually exclusive and exhaustive events, then

$$P(A) = \sum_{j=1}^{m} P(A|B_j)P(B_j) \quad (2.18)$$

$P(B) = 1 - P(A) = 0.05$

$P(E) = P(E \mid A)P(A) + P(E \mid B)P(B) = 0.095$

Exercise 2. The data collected from a patient are sent through a satellite connection to a remote system for data analysis. In this telemedicine system the probability that a bit is received incorrectly is P*e* = 0.0001. Compute the probability of error on a byte.

Now suppose that each bit is transmitted 3 times and that the receiver decode a bit following a majority rule. Compute the probability of error on the bit and on the byte.

Solution

Two events A and B are statistically independent if

$$P(A_iB_j) = P(A_i)P(B_j) \quad (2.20.a)$$

The transmitted bits are independent so the probabilities of error are

1)

$P_C = 1 - P_E = 0.9999$

on byte

$\mathrm{P}_{\mathrm{E}}^{\mathrm{BYTE}} = 1 - P_C^{BYTE} = 1 - P_C^8 = 0.0008$

2)

Repetition Code

$P_E^R = P_E^3 + 3P_E^2 P_C = 3*10\text{^}-8$

$\mathrm{P}_{\mathrm{E}}^{\mathrm{REPBYTE}} = 1 - P_C^{REPBYTE} = 1 - \left(P_C^R\right)^8 = 2.4*10\text{^}-7$

Exercise 3. A brain-computer interface (BCI) is a system that decodes brain activity into action of an external device, such as a computer or prosthesis. Consider a BCI that interpret EEG signals from a subject into four classes, *A*, *B*, *C*, *D*, that correspond to four actions of the external device (for example four directions in the control of a mouse for windows applications). Suppose that the probability of the four actions are P(*A*) = 0.4, P(*B*) = 0.3, P(*C*) = 0.15, P(*D*) = 0.15. Suppose that the probabilities of error of the BCI are P(*E*|*A*) = 0.1, P(*E*|*B*) = 0.05, P(*E*|*C*) = 0.2, P(*E*|*D*) = 0.1. Compute the probability of error for a generic action.

Solution

$$P(E) = P(E \mid A)P(A) + P(E \mid B)P(B) + P(E \mid C)P(C) + P(E \mid D)P(D) = 0.1$$

Now suppose there is a fifth class which corresponds to non-action (idle state) when the subject does not want to execute any of the four actions. The probability that the subject is in this idle state is P(*I*) = 0.5 and the probabilities of the four classes when the subject is not in the idle state are the same as before. The probability of decoding an action when the subject is in the idle state (error) is P(*F*) = 0.01 and the probability of detecting the idle state when the subject is thinking at any of the four actions is P(*G*) = 0.03. Compute the probability that the BCI decodes the idle state.

$$P_{BCI}(I) = P(I) * P(\overline{A} \mid I) + P(A) * P(I \mid A) =$$

$$P_{BCI}(I) = P(I)\left[1 - P(F)\right] + \left[1 - P(I)\right]P(G) = 0.51$$