2014 Lecture 1 Exercises

Exercise 1

$$f_{XY}(x,y) = \begin{cases} 16\frac{y}{x^3} & x>2,\ 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

a) $f_X(x) = \int_0^1 f_{XY}(x,y)\,dy = \frac{8}{x^3}$

$f_Y(y) = \int_2^\infty f_{XY}(x,y)\,dx = 2y$

b) $E[X] = \int_2^\infty x f_X\,dx = \int_2^\infty x \cdot \frac{8}{x^3}\,dx = 4$

$E[Y] = \int_0^1 y\, f_Y(y)\,dy = \int_0^1 y \cdot 2y\,dy = \frac{2}{3}$

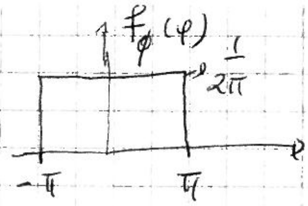
c) $f_{XY}(x,y) \stackrel{?}{=} f_X(x) \cdot f_Y(y)$

$= \frac{8}{x^3} \cdot 2y$

$= \frac{16}{x^3} y$ Yes independent

Exercise 2

$$X(t) = A\cos(2\pi f_0 t + \phi)$$



a) $$E[X(t)] = \int_{-\pi}^{\pi} X(t)\cdot f_\phi(\varphi)\, d\varphi$$

$$= \int_{-\pi}^{\pi} A\cos(2\pi f_0 t + \varphi)\cdot \frac{1}{2\pi}\, d\varphi$$

$$= \frac{A}{2\pi} \sin(2\pi f_0 t + \varphi)\Big|_{-\pi}^{\pi} = 0$$

b) $$R_{XX}(t_1, t_2) = E[X(t_1)\cdot X(t_2)]$$

$$= E[A\cos(2\pi f_0 t_1 + \phi)\cdot A\cos(2\pi f_0 t_2 + \phi)]$$

hint $\cos a\cdot\cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$

$$= A^2 E\Big[\underbrace{\frac{1}{2}\cos(2\pi f_0 (t_1 - t_2))}_{\text{constant}} + \frac{1}{2}\cos(2\pi f_0 (t_1+t_2) + 2\phi)\Big]$$

$$= \frac{A^2}{2}\cos(2\pi f_0 (t_1 - t_2)) + \frac{1}{2}\underbrace{E[\cos(2\pi f_0 (t_1+t_2) + 2\phi)]}_{\text{mean } = 0}$$

$$R_{XX}(\tau) = \frac{A^2}{2}\cos(2\pi f_0 \tau)$$

c) $$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau)\cdot e^{-j2\pi f\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} \frac{A^2}{2}\cos(2\pi f_0 \tau)\cdot e^{-j2\pi f\tau}\, d\tau$$

$$= \frac{A^2}{4}[\delta(f - f_0) + \delta(f + f_0)]$$