Lecture 3 exercises

Exercise 1

A system for home recording of ECG signals transmits the processed signals to a local hospital using a simple amplitude modulation. The transmitted signal X contains only the information on the sequence of heart beats:

$$X = \begin{cases} +A, & QRS \Rightarrow H_1 \\ 0, & No \quad QRS \Rightarrow H_0 \end{cases}$$

Where A>0.

At the receiver we have

$$Y=X+W$$

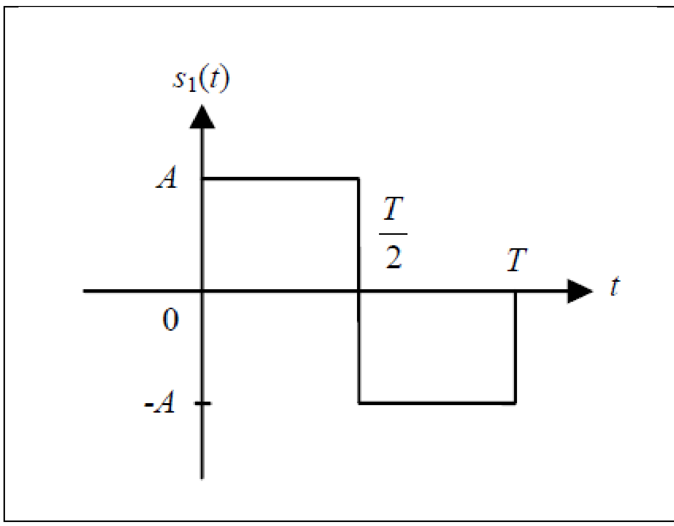
Where W is a zero-mean Gaussian random process with $\sigma^2 = \frac{A^2}{4}$. The receiver has a simple threshold algorithm that decides if the transmitted sample correspond to a QRS or not and estimate the instantaneous cardiac frequency.

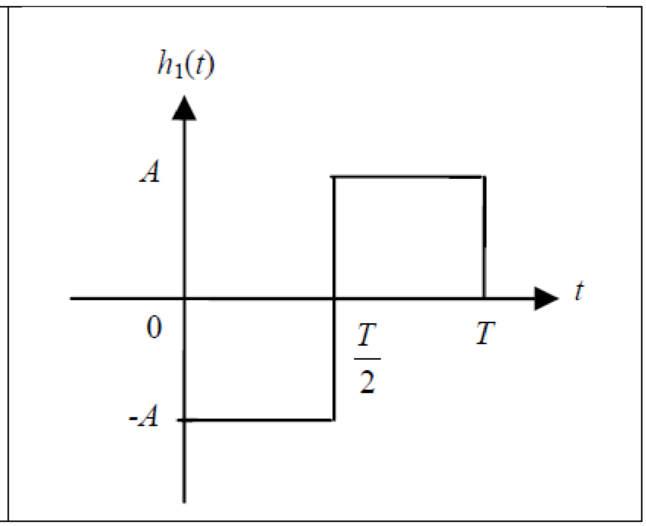
1) Find the probability density functions of Y conditioned to H_0 and H_1.
2) Calculate the log-likelihood ratio $\ln\Lambda(y) = \ln\left(\frac{f(y\,|\,H_1)}{f(y\,|\,H_0)}\right)$

Exercise 2

This is a continuation of example 10.5 in the book (Mourad Barkat page 570).

a. Show that the matched filter of $s_1(t)$ is $h_1(t)$.
b. Determine and sketch the response to $s_1(t)$ of the matched filter $h_1(t)$





Exercise 3

Consider the hypothesis testing problem in which

$$f_{Y|H_1}(y \mid H_1) = \frac{1}{2}\,\mathrm{rect}\left(\frac{y-1}{2}\right) \quad \text{and} \quad f_{Y|H_0}(y \mid H_0) = e^{-y} \ \text{for} \ y > 0$$

a) Set up the likelihood ratio test and determine the decision regions.
b) Find the minimum probability of error when $P_0 = 2/3$

Hint:

$$\mathrm{rect}(t) = \sqcap(t) = \begin{cases} 0 & \text{if } |t| > \frac{1}{2} \\ \frac{1}{2} & \text{if } |t| = \frac{1}{2} \\ 1 & \text{if } |t| < \frac{1}{2}. \end{cases}$$

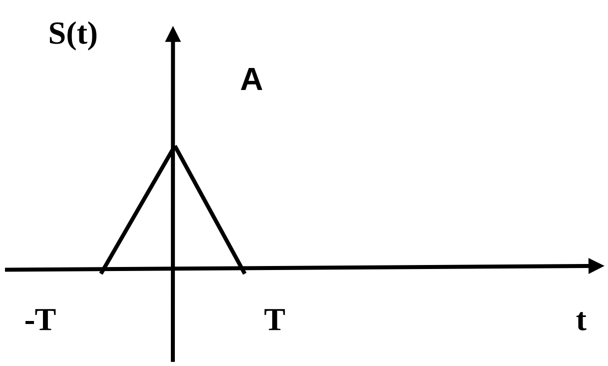
Exercise 4

Consider an electrocardiography ECG detection system.
The two hypotheses in such a system are:

H_1: ECG potential
H_0: background noise

with $P(H_1)=1/20$.

The ECG potential is approximated by a triangular pulse.



With A=1 and T=1.

Suppose the noise has a Gaussian distribution with zero mean and constant power spectrum equal to 1/10. Let the optimal matched filter be $h_{mat}(t) = s(t)$. Find the threshold for optimum detection.
Hint: The output of the matched filter is its energy at each time instant. Thus the maximum of s(t) is:

$$s_{mat}^{\max}(t) = \int_{-T}^{T} h_{mat}(t) \cdot s(t)\,dt = \int_{-T}^{T} s^2(t)\,dt = E_s$$