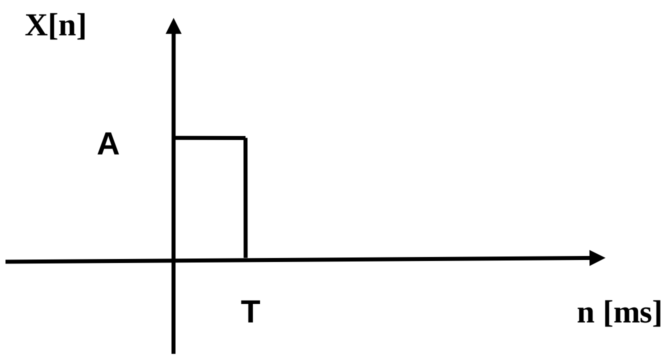
STOCHASTIC PROCESSES II

Solutions for Lecture 4

Exercise 1

We will investigate a very simple detector which is based on a threshold set on the estimate of the mean value of the signal. Consider the signal X[n] represented below:



with A=1 and T=10 ms, Fc=1000 Hz. The received signal can be modeled as following

$$Y(n) = X(n) + W(n)$$

Suppose that the noise W[n] is white, Gaussian, with zero mean and σ_W^2 =1. Demonstrate that

$$\bar{Y} = \frac{1}{N}\sum_n Y(n)$$

has $\mu_{\bar{Y}} = 1$ and in case of no event $\sigma_{\bar{Y}}^2 = \frac{\sigma_W^2}{N}$.

Solution

The mean can be easily shown that is equal to 1.

The variance of the estimator is:

$$E\{(\overline{Y}-\mu_{\overline{Y}})^2\} = E\{(\overline{Y}-\mu_X)^2\} = E\left\{(\frac{1}{N}\sum_k (X(k)+W(k)) - \mu_X)^2\right\} =$$

$$E\left\{\left(\frac{1}{N}\sum_k (X(k)+W(k))\right)^2 + \mu_X^2 - 2\mu_X \frac{1}{N}\sum_k (X(k)+W(k))\right\} =$$

$$\frac{1}{N^2}\sum_n\sum_m E\{(X(n)+W(n))(X(m)+W(m))\} + \mu_X^2 - 2\mu_X \frac{1}{N}\sum_k E\{(X(k)+W(k))\} =$$

$$\frac{1}{N^2}\sum_n\sum_m \left(E\{X(n)X(m)\} + E\{X(n)W(m)\} + E\{W(n)X(m)\} + E\{W(n)W(m)\}\right) +$$

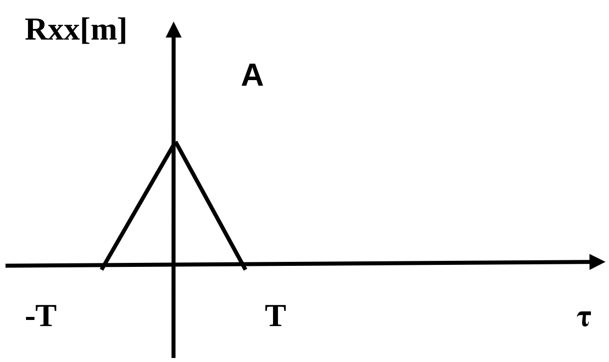
$$+\mu_X^2 - 2\mu_X \frac{1}{N}\sum_k E\{(X(k)+W(k))\} =$$

$$\frac{1}{N^2}\sum_n\sum_m \left(\mu_X^2 + E\{W(n)W(m)\}\right) + \mu_X^2 - 2\mu_X \frac{1}{N}\sum_k E\{(X(k))\} =$$

$$\frac{1}{N^2}N^2\mu_X^2 + \frac{N}{N^2}\sigma_N^2 + \mu_X^2 - 2\mu_X \frac{1}{N}N\mu_X = \frac{\sigma_N^2}{N}$$

Exercise 2

In this exercise, we will investigate the use of linear prediction for building a compression methods applied to EMG signals.
Consider the stochastic process S[n] that models an EMG signal recording. S[n] is zero mean and has autocorrelation function Rxx[m], represented below



with A=1 and T=10 ms, Fc=1000 Hz and zero mean.

1) Find the linear minimum mean squared predictor of S[n] based on the preceding N=2 samples.

When predicting X[k], the two preceding samples are n = k-2, k-1, Thus

$$\hat{X}[k] = h_0 \cdot X[k-2] + h_1 \cdot X[k-1]$$

Using Minimization of the MSE we have:

For i = k-2 => *h_0*Rxx(k-2-k+2) + h_1*Rxx(k-1-k+2) = Rxx(k-k+2)*
For i = k-1 => *h_0*Rxx(k-2-k+1) + h_1*Rxx(k-1-k+1) = Rxx(k-k+1)*

Solving the system with equations

*h_0*Rxx(0) + h_1*Rxx(1) = Rxx(2)*
*h_0*Rxx(1) + h_1*Rxx(0) = Rxx(1)*

we have

$$h_0 = -0.05256$$
$$h_1 = 0.9474$$

2) Find the mean and variance of the error when using the linear predictor developed in point 1.

mean

$$E\left\{\left(X[k] - \hat{X}[k] \right)\right\} = E\{(X[k] - h_1 X[k-1] - h_2 X[k-2])\} = 0$$

var*iance*

$$E\left\{\left(X[k] - \hat{X}[k] \right)^2\right\} = E\{(X[k] - h_1 X[k-1] - h_2 X[k-2])^2\} =$$

$$E\{X^2[k] + h_1^2 X^2[k-1] + h_2^2 X^2[k-2] - 2h_1 X[k]X[k-1] - 2h_2 X[k]X[k-2] + 2h_1 h_2 X[k-1]X[k-2]\} =$$

$$R_{XX}[0] + h_1^2 R_{XX}[0] + h_2^2 R_{XX}[0] - 2h_1 R_{XX}[1] - 2h_2 R_{XX}[2] + 2h_1 h_2 R_{XX}[1] = 0.1895$$