AALBORG UNIVERSITY

Welcome to Time-Frequency Analysis, Adaptive Filtering and Source Separation

Lecture 4: Continuous Wavelet Transform

Ernest N. Kamavuako

Power spectra

- Classical methods
- Modern approaches
  - Parametric methods
  - Non parametric
- Time Frequency analysis
  - Time-frequency methods
  - Time-scale methods (**WAVELET**)

Classical method

Fourier transform

- Very robust spectral estimators
- Almost no assumption about the nature of the data
- The waveform outside the data window is assumed to be zero
  - Leading to distortionin in the estimate
  - The window introduces distortion

Modern approaches

- ✓ Design to overcome some distortion problem especially for short data segments
- ✓ Take advantage of knowledge about the source of signal

<u>Model-based (parametric):</u>

- ✓ Model the data outside the waveform
  - ✓ Eliminates the need for windowing
  - ✓ Can improve resolution and fidelity especially with high level of noise
- ✓ Performance depends too much on the model
- ✓ AR, MA, ARMA

Modern approaches

Non parametric:

- ✓ Eigen analysis frequency estimation
- ✓ If noise is not white => Problem
- ✓ Bad in estimating broadband process

Do not concern about timing
Works best for stationary waveforms

Time-frequency analysis

<u>Short-time Fourier transform: THE SPECTROGRAM</u>

- ❑ How to select the appropriate window length?
- ❑ Time-frequency trade-off

<u>Wigner-Ville distribution</u>

One of the best studied and best understood TF methods developed by Wigner to use in Physics and applied to signal processing by Ville.

Time-frequency analysis

Wigner-Ville distribution

- Use of instantaneous autocorrelation function preserving time information
- Cross-products or cross terms

Cohen's class

- Filtering of the WV distribution using Kernels
- Good picture of TF structure

Uncertainty principle

$$\sigma^2_\omega * \sigma^2_t = 1/4$$

Wavelet transform

- None of the methods developed so far has solved the time-frequency problem without constraints or added artifacts.

- Fourier transform
- $F(m) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j2\pi mt} dt$

- A sine wave is perfectly localized in frequency, but infinite in time.

Short-time fourier transform

$$STFT(t,f) = \int_{-\infty}^{+\infty} x(\tau) w(t-\tau) e^{-j2\pi f t} d\tau$$

In general

$$X(t,m) = \int_{-\infty}^{+\infty} x(\tau) \cdot f(t-\tau)_m d\tau$$

In the wavelet transform (WT), family members consist of enlarged and compressed versions of the basic function

Wavelet transform

From french: ondelette (small wave)

- Finite in time
- $W(a,b) = \int_{-\infty}^{+\infty} x(t) \cdot \frac{1}{\sqrt{a}} \psi^* \left(\frac{t-b}{a}\right) \mathrm{dt}$
- Different values of *a* and *b* gives a serie of wavelets that may be addedd together to reconstruct the signal
- ***They are all localized in both time and frequency, but not precisely localized in either.***