AALBORG UNIVERSITY

Welcome to Time-Frequency Analysis, Adaptive Filtering and Source Separation

Lecture 6: Filter Banks Wavelet Packet and Parameterization

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From surface to deep learning

Storyline

Questions and Answers

Continuous Wavelet Transform (CWT)

From french: ondelette (small wave)

- Finite in time
- $W(a,b) = \int_{-\infty}^{+\infty} x(t) \cdot \frac{1}{\sqrt{a}} \psi^* \left(\frac{t-b}{a}\right) \mathrm{dt}$
- Different values of *a* and *b* gives a serie of wavelets that may be addedd together to reconstruct the signal
- ***They are all localized in both time and frequency, but not precisely localized in either.***

Continuous Wavelet Transform (CWT)

CWT

- $W(a,b) = \int_{-\infty}^{+\infty} x(t) \cdot \frac{1}{\sqrt{a}} \psi^{*}\left(\frac{t-b}{a}\right) \mathrm{dt}$

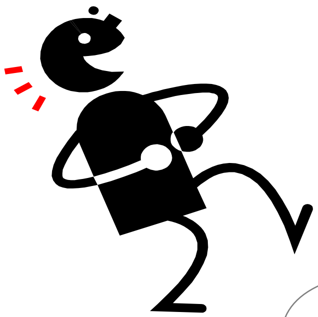
iCWT

- $x(t) = \frac{1}{C} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W(a,b) \cdot \psi^{*}(t) \mathrm{d}a db$

Discrete Wavelet Transform (DWT)

DFT and CFT

Why *CWT* and *DWT*?

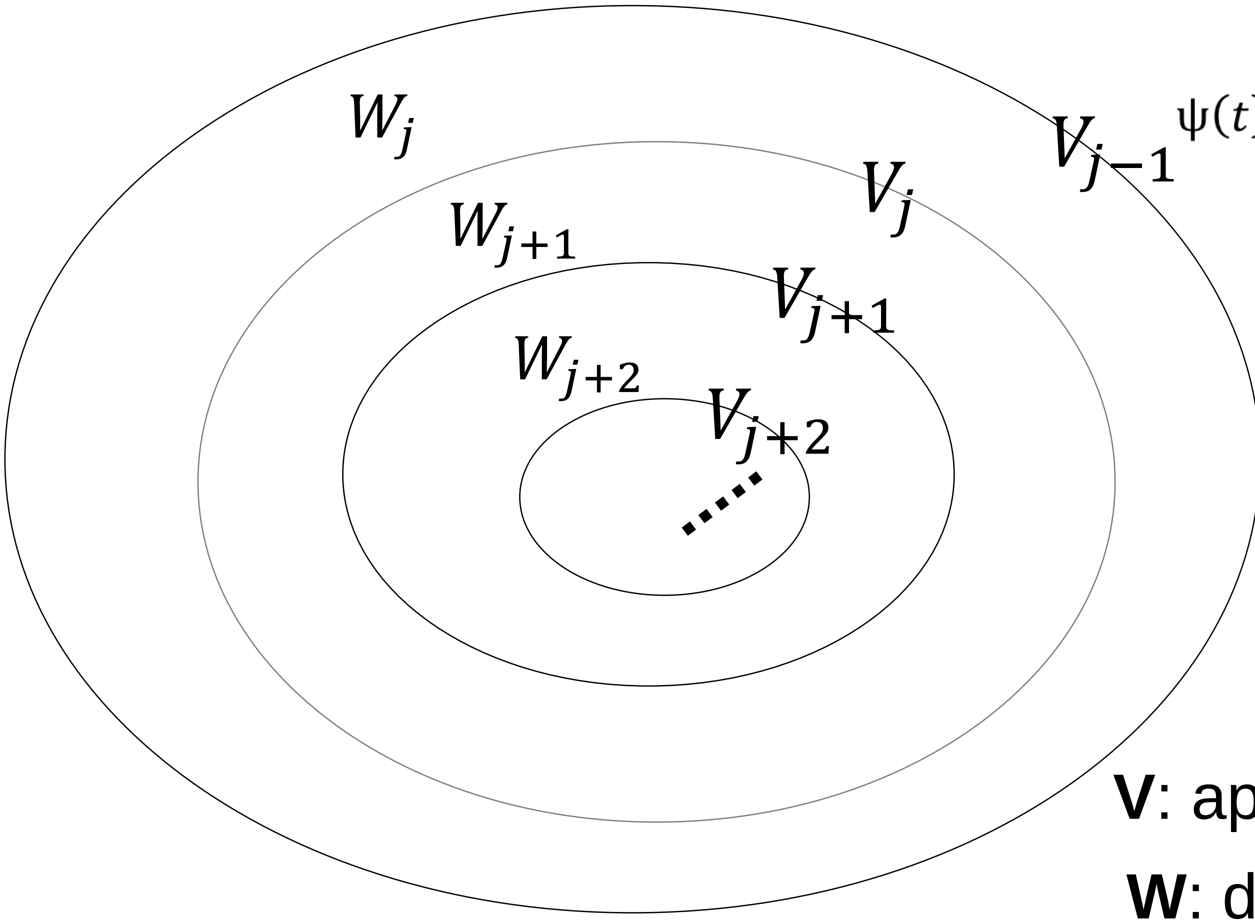


CWT: Any value of *a* and *b*.
DWT: **Choose only values so that <u>non redudant and invertible</u>.**
DFT: 1-D, but DWT: 2-D

Multiresolution Analysis

$$\theta(t) = \sum_{n=-\infty}^{+\infty} h_n \cdot \sqrt{2}\,\theta(2t-n)$$

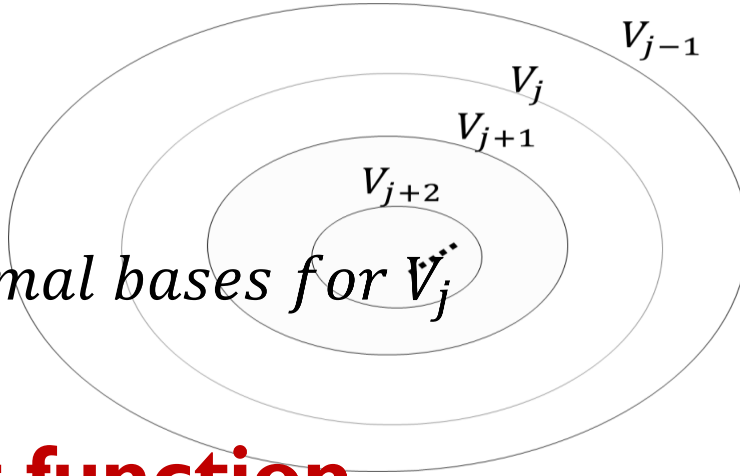
$$\psi(t) = \sum_{n=-\infty}^{+\infty} g_n \cdot \sqrt{2}\,\theta(2t-n)$$



V: approximation space

W: detail space

Multiresolution Analysis



$$\left\{2^{-\frac{j}{2}} \cdot \theta\left(2^{-j}t - n\right)\right\} \; as\ orthonormal\ bases\ for\ V_j$$

$\theta(t)$ is called Scaling function

$$V_{j-1} = \bigcup_{k=0}^{+\infty} W_{j+k}$$

$$\left\{2^{-\frac{j}{2}} \cdot \psi\left(2^{-j}t - n\right)\right\} \; as\ orthonormal\ bases\ for\ W_j$$

$\psi(t)$ is called wavelet function

Discrete Wavelet transform

$$\beta_{n,j} = \int_{-\infty}^{+\infty} f(t) \cdot \frac{1}{\sqrt{2^j}} \psi^* \left(\frac{t - 2^j n}{2^j} \right) \mathrm{dt}$$

$$W(a,b) = \int_{-\infty}^{+\infty} f(t) \cdot \frac{1}{\sqrt{a}} \psi^* \left(\frac{t - b}{a} \right) \mathrm{dt}$$

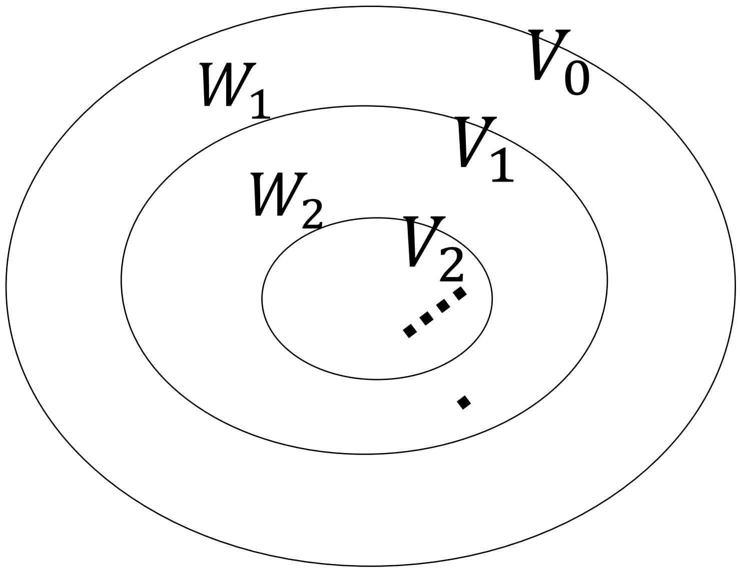
$a = 2^j$ and $\mathrm{b} = 2^j n$: Dyadic wavelet transform

Filter Banks

- A filter bank is an array of band-pass filters that separates the input signal into multiple components, each one carrying a single frequency subband of the original signal.
- We have seen that multiresolution Analysis allows us to decompose a signal into approximations and details.
- Filter Bank is a way to implement the MRA and DWT.

Filter Banks

$$P_{V_0}f = \sum_n c_n \theta(t-n) = f(t)$$

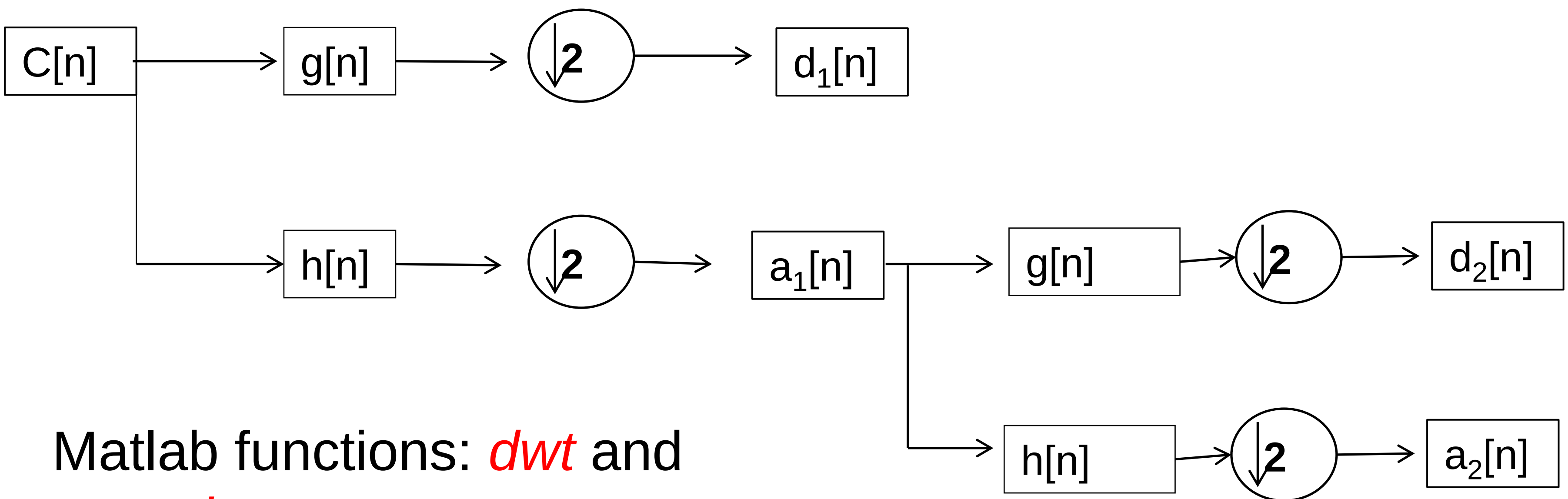


$$P_{V_1}f = \sum_k a_k \frac{1}{\sqrt{2}} \theta\left(\frac{t}{2} - n\right)$$

$$P_{W_1}f = \sum_k d_k \frac{1}{\sqrt{2}} \Psi\left(\frac{t}{2} - n\right)$$

We would like to find a_k and d_k, not by using $f(t)$ but its representation in $V_0(c_n)$. a_k, d_k?

Analysis: from fine scale to coarser scale

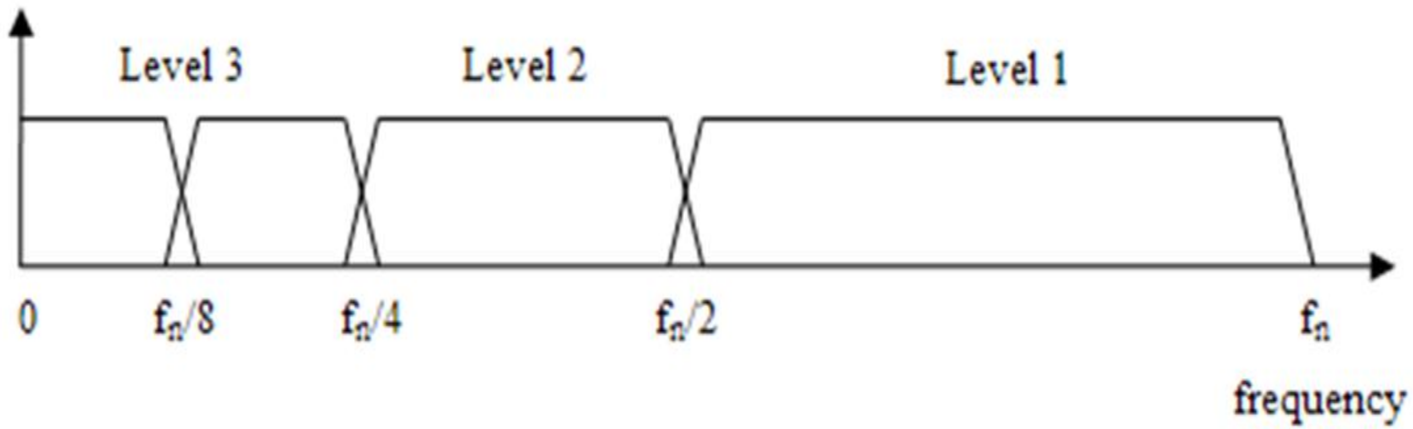


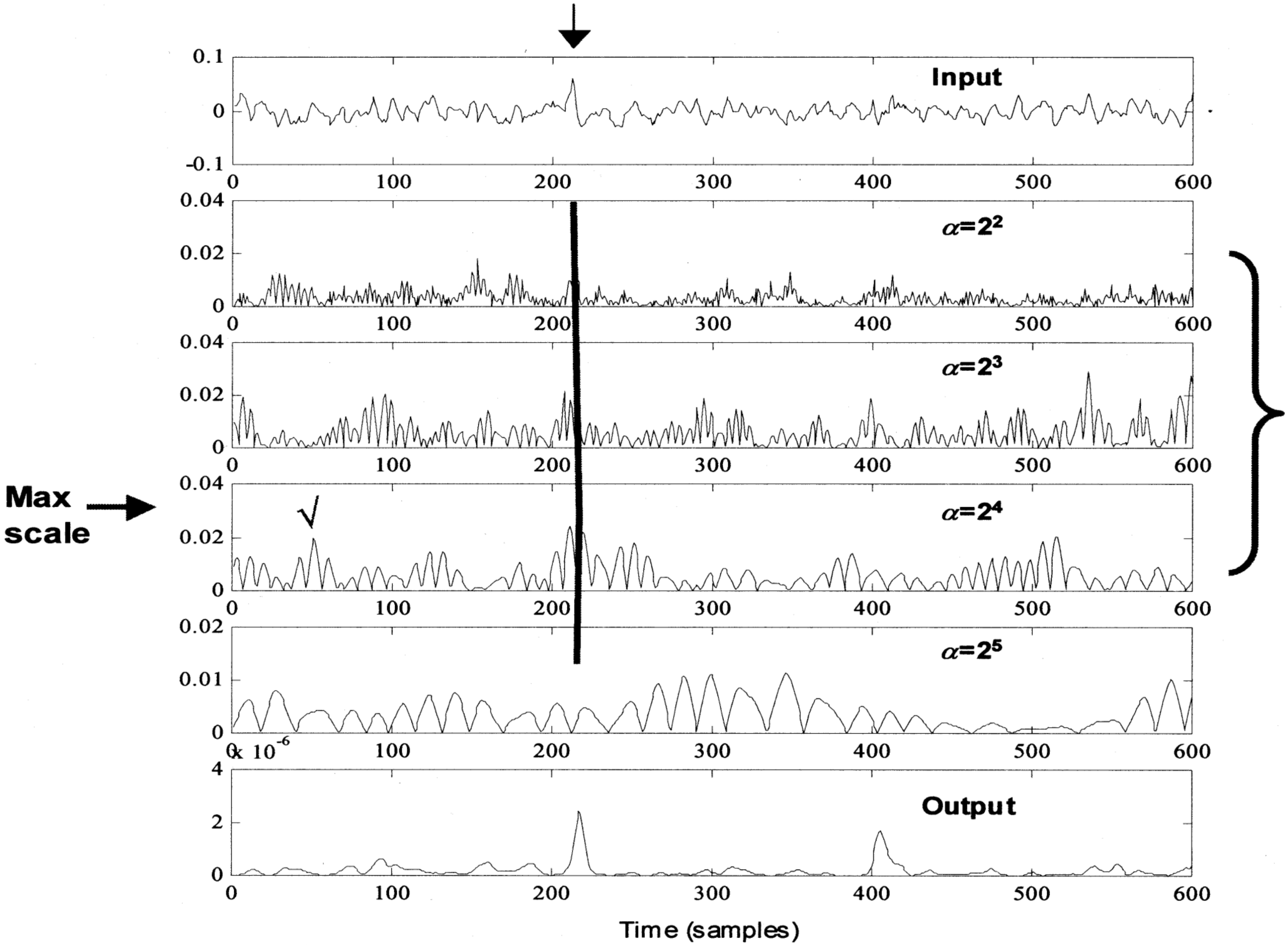
Matlab functions: *dwt* and *wavedec*

[cA, cD] = dwt(x, Lo, Hi);
= dwt(x, 'wname');

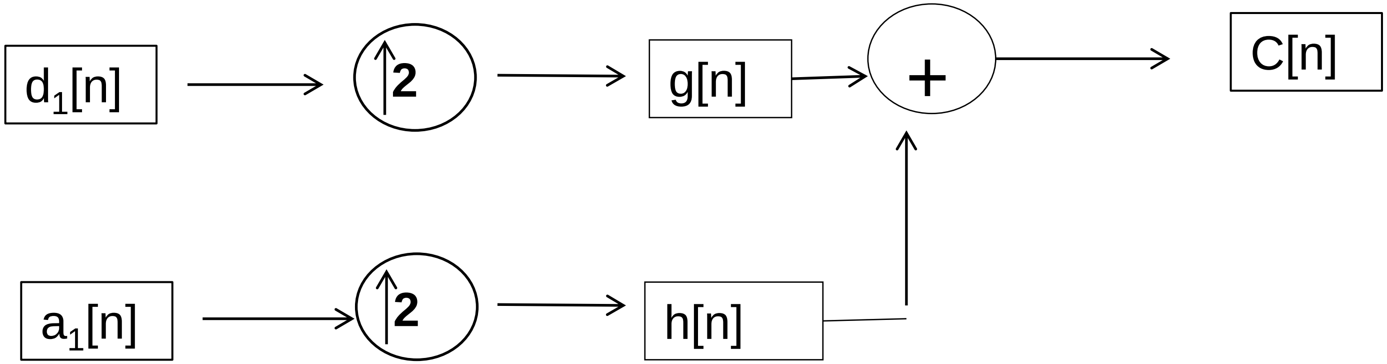
[C, L] = wavedec(x, N, Lo, Hi);
= wavedec(x, N, 'wname');

Analysis: from fine scale to coarser scale




Input
α=2²
α=2³
α=2⁴
α=2⁵
Output
Max scale
x 10⁻⁶
Time (samples)

Synthesis: from coarse scale to fine scale

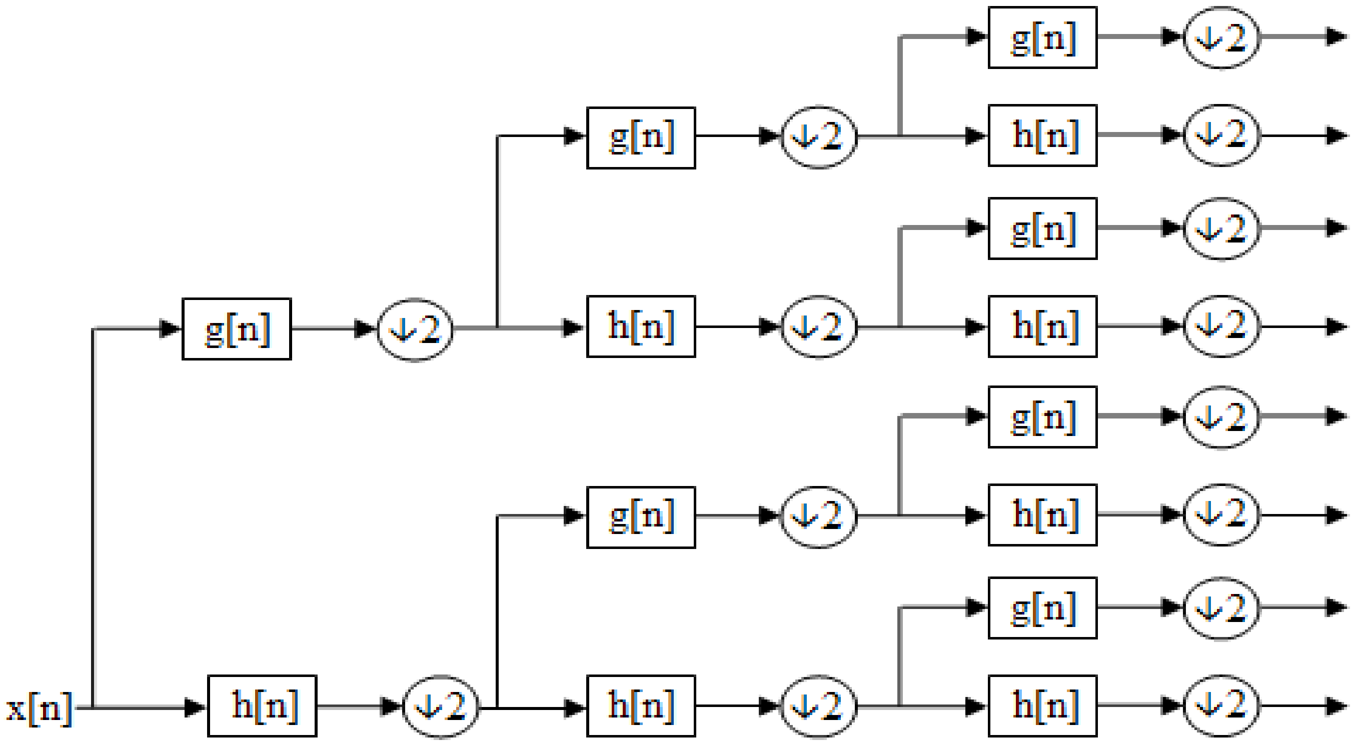


Matlab functions: *idwt* and *waverec*

```
x = idwt(cA, cD, Lo, Hi);
  = idwt(cA, cD, 'wname');
```

```
x   = waverec(C, L, Lo, Hi);
    = waverec(C,L, 'wname');
```

Wavelet Packet



Wavelet Parameterization

- WT requires the selection of the mother wavelet.
- Wavelet usually designed similar to the signal.
- Here The mother wavelet is parameterized.
- ψ is defined by a low-pass filter *h* and its associated high-pass filter *g*.

$$h[0] = (1 - \cos(\alpha) + \sin(\alpha)) / (2\sqrt{2})$$
$$h[1] = (1 + \cos(\alpha) + \sin(\alpha)) / (2\sqrt{2})$$
$$h[2] = (1 + \cos(\alpha) - \sin(\alpha)) / (2\sqrt{2})$$
$$h[3] = (1 - \cos(\alpha) - \sin(\alpha)) / (2\sqrt{2})$$

Wavelet Parameterization

$$h[0] = (1 - \cos(\alpha) + \sin(\alpha)) / (2\sqrt{2})$$
$$h[1] = (1 + \cos(\alpha) + \sin(\alpha)) / (2\sqrt{2})$$
$$h[2] = (1 + \cos(\alpha) - \sin(\alpha)) / (2\sqrt{2})$$
$$h[3] = (1 - \cos(\alpha) - \sin(\alpha)) / (2\sqrt{2})$$

$$g[n] = (-1)^{1-n} h[1-n]$$

If $\alpha = 0,\ h = \left[0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right] \Rightarrow \mathrm{g} = \left[0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right]$

[h,g] = wfilters('db2') **Flip *h* and change signs of odd values**

$$h = [-0.1294, 0.2241, 0.8365, 0.4830],$$
$$\mathrm{g} = [-0.4830, 0.8365, -0.2241, -0.1294]$$