**5.1.**

$$y[n] = \begin{cases} 1, & 0 \le n \le 10, \\ 0, & \text{otherwise} \end{cases}$$

Therefore,

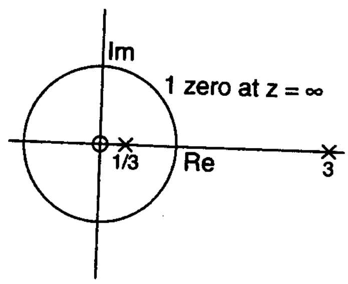
$$Y(e^{j\omega}) = e^{-j5\omega} \frac{\sin \frac{11}{2}\omega}{\sin \frac{\omega}{2}}$$

This $Y(e^{j\omega})$ is full band. Therefore, since $Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$, the only possible $x[n]$ and $\omega_c$ that could produce $y[n]$ is $x[n] = y[n]$ and $\omega_c = \pi$.

**5.2.** We have $y[n-1] - \frac{10}{3}y[n] + y[n+1] = x[n]$ or $z^{-1}Y(z) - \frac{10}{3}Y(z) + zY(z) = X(z)$. So,

$$\begin{aligned} H(z) &= \frac{1}{z^{-1} - \frac{10}{3} + z} \\ &= \frac{z}{(z-\frac{1}{3})(z-3)} \\ &= \frac{-\frac{1}{8}}{z-\frac{1}{3}} + \frac{\frac{9}{8}}{z-3} \end{aligned}$$

(a)




(b)

$$H(z) = \frac{-\frac{1}{8}z^{-1}}{1-\frac{1}{3}z^{-1}} + \frac{\frac{9}{8}z^{-1}}{1-3z^{-1}}$$

Stable ⇒ ROC is $\frac{1}{3} \le |z| \le 3$. Therefore,

$$h[n] = -\frac{1}{8}\left(\frac{1}{3}\right)^{n-1} u[n-1] - \frac{9}{8}(3)^{n-1}u[-n]$$

$$y[n-1] + \frac{1}{3}y[n-2] = x[n]$$

$$z^{-1}Y(z) + \frac{1}{3}z^{-2}Y(z) = X(z)$$

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} + \frac{1}{3}z^{-2}} \\ H(z) &= \frac{z}{1+\frac{1}{3}z^{-1}} \end{aligned}$$

i) $\frac{1}{3} < |z|$, $h[n] = (-\frac{1}{3})^{n+1}u[n+1] \Rightarrow$ answer (a)
ii) $\frac{1}{3} > |z|$,

$$\begin{aligned} h[n] &= -\left(-\frac{1}{3}\right)^{n+1} u[-n-2] \\ &= -\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)^{n} u[-n-2] \\ &= \frac{1}{3}\left(-\frac{1}{3}\right)^{n} u[-n-2] \Rightarrow \text{answer (d)} \end{aligned}$$

**5.4.** (a)

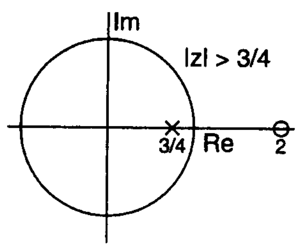
$$x[n] = \left(\frac{1}{2}\right)^n u[n] + (2)^n u[-n-1]$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{z}{z-2}, \quad \frac{1}{2} < |z| < 2$$

$$y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n]$$

$$Y(z) = \frac{6}{1-\frac{1}{2}z^{-1}} - \frac{6}{1-\frac{3}{4}z^{-1}}, \quad \frac{3}{4} < |z|$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-\frac{3}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})} \cdot \frac{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}{-\frac{3}{2}z^{-1}} = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}, \quad |z| > \frac{3}{4}$$




(b)

$$H(z) = \frac{1}{1-\frac{3}{4}z^{-1}} - \frac{2z^{-1}}{1-\frac{3}{4}z^{-1}}, \quad |z| > \frac{3}{4}$$

$$h[n] = \left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{3}{4}\right)^{n-1} u[n-1]$$

(c)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}$$

$$Y(z) - \frac{3}{4}z^{-1}Y(z) = X(z) - 2z^{-1}X(z)$$

$$y[n] - \frac{3}{4}y[n-1] = x[n] - 2x[n-1]$$

(d) The ROC is outside $|z| = \frac{3}{4}$, which includes the unit circle. Therefore the system is stable. The $h[n]$ we found in part (b) tells us the system is also causal.

**5.5.**

$$y[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n] + u[n]$$

$$Y(z) = \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-z^{-1}}, \qquad |z| > 1$$

$$x[n] = u[n]$$

$$X(z) = \frac{1}{1-z^{-1}}, \qquad |z| > 1$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 - \frac{19}{6}z^{-1} + \frac{2}{3}z^{-2}}{1 - \frac{7}{12}z^{-1} + \frac{1}{12}z^{-2}}, \qquad |z| > \frac{1}{3}$$

(a) Cross multiplying and equating $z^{-1}$ with a delay in time:

$$y[n] - \frac{7}{12}y[n-1] + \frac{1}{12}y[n-2] = 3x[n] - \frac{19}{6}x[n-1] + \frac{2}{3}x[n-2]$$

(b) Using partial fractions on $H(z)$ we get:

$$H(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{z^{-1}}{1-\frac{1}{3}z^{-1}} + \frac{1}{1-\frac{1}{4}z^{-1}} - \frac{z^{-1}}{1-\frac{1}{4}z^{-1}} + 1, \qquad |z| > \frac{1}{3}$$

So,

$$h[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{3}\right)^{n-1} u[n-1] + \left(\frac{1}{4}\right)^n u[n] - \left(\frac{1}{4}\right)^{n-1} u[n-1] + \delta[n]$$

(c) Since the ROC of $H(z)$ includes $|z| = 1$ the system is stable.

**5.6.** (a)

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}(2)^n u[-n-1]$$

$$X(z) = \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{4}{3}}{1-2z^{-1}} = \frac{1}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}, \qquad \frac{1}{2} < |z| < 2$$

(b)

$$Y(z) = \frac{1-z^{-2}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

This has the same poles as the input, therefore the ROC is still $\frac{1}{2} < |z| < 2$.

(c)

$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-2} \Leftrightarrow h[n] = \delta[n] - \delta[n-2]$$