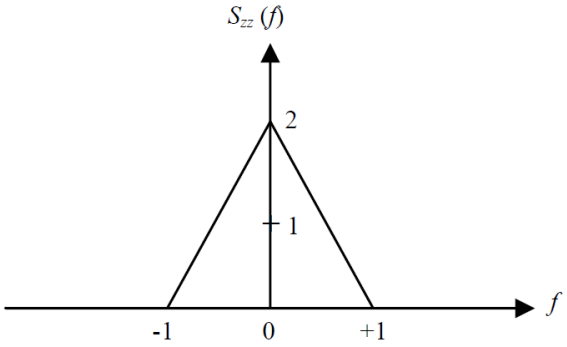
Solutions Lec 6

1.

3.11 The autocorrelation function of the process $Z(t) = Y(t) - X(t-1)$ is

$$\begin{aligned} R_{zz}(\tau) &= E\{[Y(t+\tau) - X(t+\tau-1)][Y(t) - X(t-1)]\} \\ &= R_{yy}(\tau) - R_{yx}(\tau+1) - R_{xy}(\tau-1) + R_{xx}(\tau) \\ &= R_{yy}(\tau) + R_{xx}(\tau) \end{aligned}$$

since $R_{yx} = R_{xy} = 0$ from orthogonality. Therefore, $S_{zz}(f) = 2S_{yy}(f) = 2S_{xx}(f)$ as shown below.

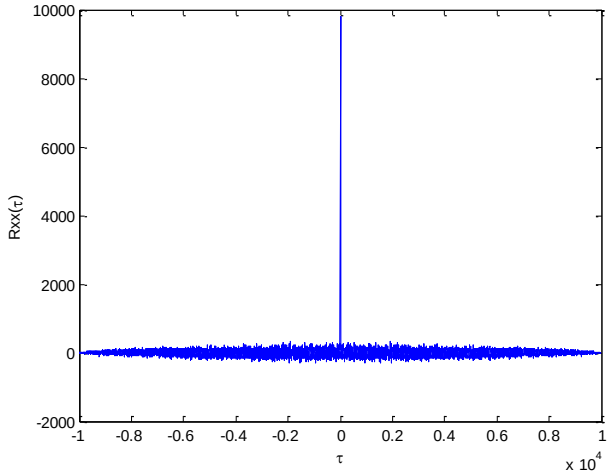


2.

The autocorrelation is the invers Fourier transform of the power spectrum. The invers Fourier transform of $Sxx(f) = 1$ is:

$$Rxx(\tau) = \int_{-\infty}^{\infty} i\, e^{j2\pi f\tau} dw = \delta(\tau)$$

Obs: the exercise is updated

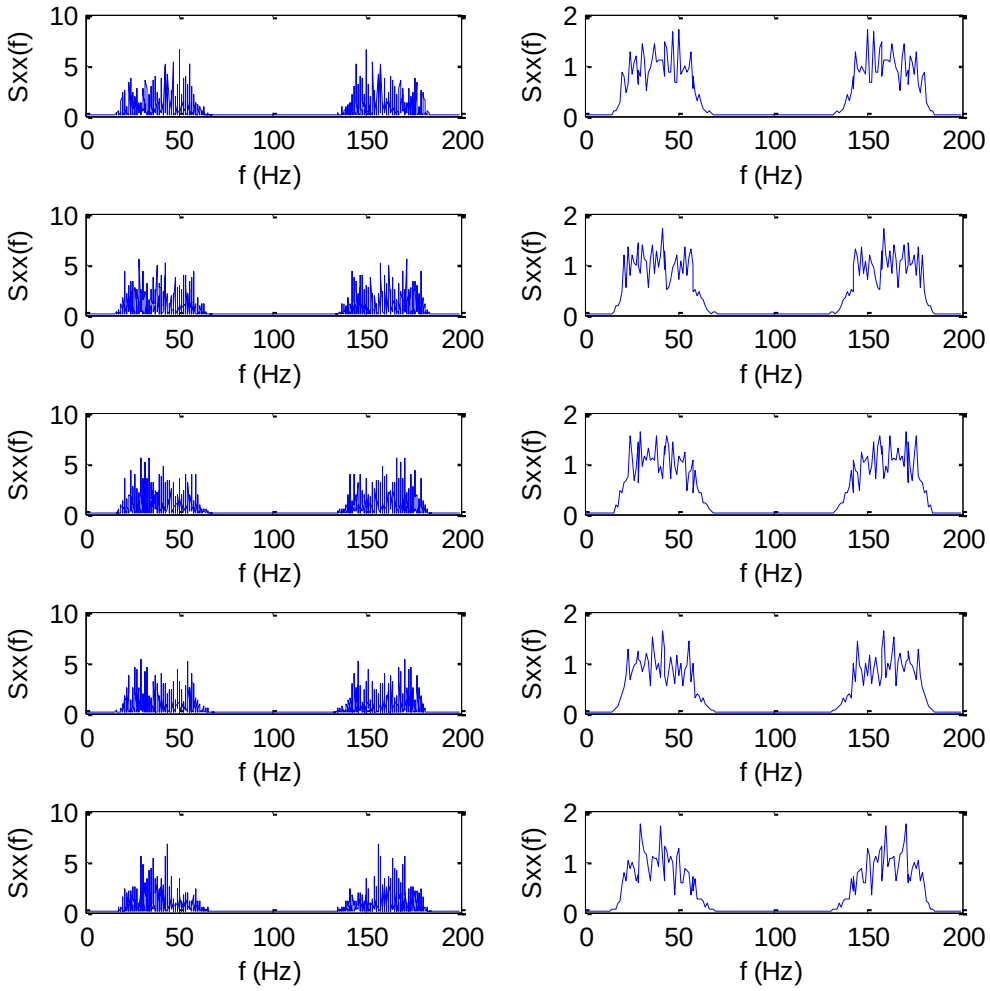


As seen does the autocorrelation include a impuse at lag 0 and it approcese zero at other lag values.

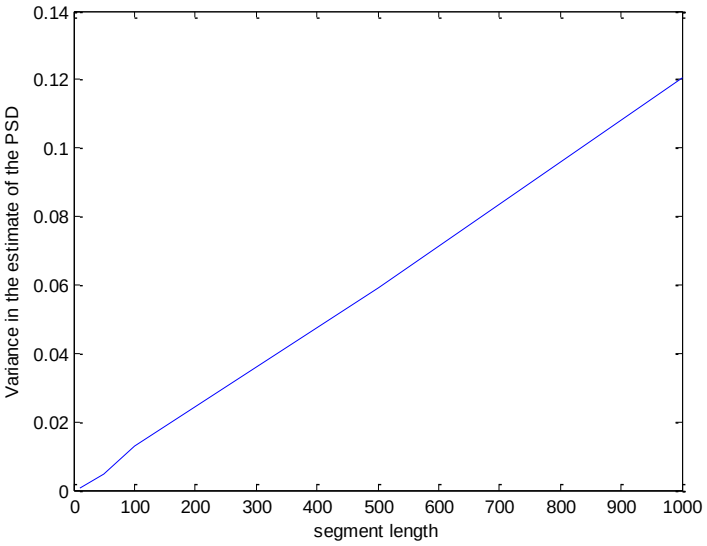
3.

See Matlab code pgram.m and apgram.m and make5PSDs.m

PSD of 5 different realizations of the process.



The average variance is approximately 0.3 for the normal periodogram and 0.03 for the average periodogram, see matlab results



The figure shows that as the segment length decrease (the number of segments increases) the variance decreases.