Energy and Power in (Sports) Biomechanics

Objectives

- Review and learn basic energy formulae
- Determine total body energy by centre of mass calculations
- The use of power equations in performance assessment
- Apply energy estimations to an example of changes to sport equipment
- Learn about basic physics principles of rotational movements

Contents

1. Calculation of total body energy
2. Estimates of rotational energy
3. Estimates of energy consumption with varied running footwear
4. Energy calculations in high jump
5. Rotational movements
6. Power calculations in jump tests
Energy in a purely ‘physics’ sense

Mechanical energy:

\[ E_{\text{kin}} = 0.5 \ m \ v^2 \]
\[ E_{\text{pot}} = m \ g \ h \]
\[ E_{\text{rot}} = 0.5 \ I \ \omega^2 \]
\[ E_{\text{el}} = 0.5 \ k \ \Delta x^2 \]

Total body energy:

\[ E_{\text{tot}} = \sum_{i=1}^{n} E_{\text{kin} \ i} + E_{\text{pot} \ i} + E_{\text{rot} \ i} \ n \ = \ \# \ of \ segments \]

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CoM velocity?

\[ E_{\text{kin \ tot}} = \sum m_i \ v_i^2 \]

+ potential Energy

\[ E_{\text{pot}} = \sum m_i \ g \ h_i \]

Question: Is the total Energy equal to CoM based calculations?

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COM calculation from segment positions

\[ x_0 = \frac{\sum m_i \ x_i}{M} \]
\[ y_0 = \frac{\sum m_i \ y_i}{M} \]
Rotational Energy contribution?

\[ E_{\text{rot tot}} = \Sigma I_\omega \omega^2 \]

**Example**

Leg: \( m_{\text{leg}} = 0.0465 \text{ BM} \)

\( V_{\text{leg sprint}} = 20 \text{ m/s} \)

\( l_{\text{leg}} = 0.064 \text{ kg m}^2 \)

\( \omega_{\text{leg sprint}} = 500 \degree/\text{s} \)

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**Mechanics**

Hook’s law:

Force is proportionate to deformation

- deformation: \( s \text{ [m]} \)
- force: \( F \text{ [N]} \)
- stiffness: \( k \text{ [N/m]} \)

Hook’s:

\[ F_s = -k \cdot s \]

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**Example: Running with and without shoes**

Oxygen consumption: + 4 - 5% when running with shoes

(6 - 7 min in a marathon)

- Total work: \( 10^7 \text{ J} \)
- Shoe mass: 100 g
- Lifting height: 0.2 m
- Maximal speed of foot during swing: 10 m/s
- Step length: 2 m \( \Rightarrow \) 20 ksteps

Additional work due to gravity

Additional work to accelerate the shoe
Application to high jump

In high jump the initial energy has to be transformed into jump energy.

There is no linear relationship between approach velocity and vertical take-off velocity. (Dopena et al., 1990)

Transformation is coupled with a decrease in total energy during the last ground contact. (Brügemann & Arampatzis, 1991)
Purpose

- To examine the approach and take-off strategies of high jumpers at the world class level.
- To determine how to estimate the optimal take-off behavior from given initial characteristics.

Methods

Data Acquisition and Analysis

- four stationary PAL video cameras (50 Hz), two for each side, synchronised
- calibration with $2 \times 2 \times 3$ m cube
- digitisation with Peak Motus (19 points)
- calculation of body angles, CM position, CM velocity and total energy by "fast information program" using DLT
Calculations

(1) \[ H = H_0 + \frac{V^2 \sin^2 \alpha}{2g} \]

(2) \[ H = \frac{E_{p2}}{g} + \frac{E_{k2} \sin^2 \alpha}{2g} \]

where:

\[ E_{k2} = \frac{E_{kin}^2}{m} \]

\[ E_{p2} = \frac{E_{pot}^2}{m} \]

\[ T_{in} = \frac{\alpha}{E_{dec}} \]
The effective height \( H \) can be expressed as a function of:
initial energy, energy loss and
transformation index.

\[
H = \frac{E_p^2}{g} + \frac{(E_{T1} - E_{\text{dec}} - E_{p2}) \sin^2 (T_\text{in} E_{\text{dec}})}{g}
\]

(4)

RESULTS

Cluster Analysis
Two groups were identified that showed both varying initial conditions and jumping strategies.
Group 1 showed a lower horizontal velocity and therefore, a lower initial energy prior to the last ground contact.

Group 2 had a markedly increased initial energy and demonstrated a lower transformation index.

Formula: $T_{in} = \exp(-c \cdot E_{dec})$

$R^2 = 0.98$

With that:
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Group1 (n=16)</th>
<th>Group2 (n=10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal TD velocity</td>
<td>7.11 (0.29)</td>
<td>7.83 (0.43)</td>
</tr>
<tr>
<td>Decrease in h. velocity</td>
<td>3.22 (0.30)</td>
<td>4.05 (0.56)</td>
</tr>
<tr>
<td>Horizontal TO velocity</td>
<td>3.89 (0.27)</td>
<td>3.77 (0.22)</td>
</tr>
<tr>
<td>Vertical TO velocity</td>
<td>4.44 (0.11)</td>
<td>4.29 (0.26)</td>
</tr>
</tbody>
</table>

**Discussion**

Both of the identified strategies resulted in comparable effective heights.

The faster approaching athletes could not benefit from their higher energy produced during the run-up.
• Since body orientation at touch-down shows no significant differences a possible explanation could be an altered muscle stiffness.

• The influence of changes in muscle stiffness has to be further investigated.

• For both groups the optimum energy loss that leads to the maximum vertical velocity after the take-off can be estimated.

\[
\begin{array}{c|c}
\text{Jump height (m)} & \\
\hline
\text{Sotomayor } E_{\text{dec}} & 41.07 \text{ J/kg} \\
\text{Partyka } E_{\text{dec}} & 36.53 \text{ J/kg} \\
\text{Forsyth } E_{\text{dec}} & 35.82 \text{ J/kg} \\
\text{Matusevitch } E_{\text{dec}} & 36.65 \text{ J/kg} \\
\end{array}
\]
Training concepts may be developed to increase performance at both the world class level and in sub-elite jumpers.

Summary
Relationships between kinematic parameters and jump performance are usually not very strong. Therefore, predictions are almost impossible. Energy approach provides much simpler analysis and can be used on a daily basis (?). Muscle stiffness/joint stiffness has to be included.
Initial knee angle, incoming speed and jump height

Results from simulation study (Alexander, 1980)

Some basic rules on rotation

1. Gathering angular momentum
2. Regulating rotations
3. Aerials
   Summersaults with twist
4. Assessment of jump performances
   - simple tests
   - using a force platform

Aerials

Off-centre force during take-off creates torque. The amount of angular momentum in the airborne phase cannot be changed.

Resulting in:
Momentum: \( M = m \cdot v \) (Impulse)
Angular Mom.: \( A = I \cdot \omega \)
How does a snowboarder/skier produce angular momentum?

Virtually no friction to generate $F_{rot}$!

From TO angular momentum is preserved; CoM follows parabolic path (i.e., projectile motion)

Regulation of rotation

Athletes have to change body configuration to alter $I$ (Moment of Inertia); Angular Momentum unchanged when airborne:

$A = I \cdot \omega \rightarrow$ reducing $I$ increases $\omega$.

Used for slowing down or accelerating rotations.

How is forward rotation generated in ski jumps?

Aerials with twist

If body position changes body rotates about a different local axis. Angular velocity is a vector along the rotational axis (right hand rule)
Result
(Yeadon 2000, etc.)
a) in athlete's coordinate system
b) in laboratory coordinate system

Jump Performance
Common tests:
- Jump and reach
- Assumptions: ...
- Jump belt test:
  Assumptions: ...

Determining Jump Height
Common tests
- Determine flight time:
  (contact mats or foot switches, video?)
  Assumptions: ...

signal

v

flight time

time
Impulse-Momentum Relationship

Momentum (of a moving object):
\[ \text{Mom} = m \times v(t) \]
at any point in time \( t \)

Impulse: \( I = F \times t; \)
actually: \( I = \int F \times dt \)

In other terms

\( BW_{\text{impulse}}: BWI = t_{\text{contact}} \times BW \)

Total Impulse: \( TI = \int F(t) \, dt = \sum F_i \times Dt \)

Resulting momentum:
\[ \text{Mom} = TI - BWI \]

Realisation in excel

To calculate from measured force signal:
- Acceleration of CoM
- Velocity of CoM
- Displacement of CoM
  (all in vertical (z) direction; however applies for horizontal forces as well)

..\Labs\Jumps_Template.xls
What else to get from GRF

\[ F = m \cdot a \]

\[ v(t) = v_0 + \int a \, dt \]

\[ s(t) = s_0 + \int v \, dt \]

\[ s(t) = s_0 + v_0 (t-t_0) + \int \int a \, dt^2 \]

Example:

**Result**

**Counter Movement Jump**

- **Msubj (kg)**: 81.00
- **hBox (m)**: 0.00
- **tcont (s)**: 0.70
- **Fmax (N)**: 2474.45
- **vTD (m/s)**: 0.00
- **vTO (m/s)**: 3.37
- **maxP (W/kg)**: 74.00
- **hj (m)**: 0.57
- **neg Work (J/kg)**: -2.80
- **pos Work (J/kg)**: 7.11
- **gain (J/kg)**: 9.71